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Wilson Lines in Warped Space: Dynamical Symmetry Breaking and Restoration

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Abstract

The dynamics of Wilson lines integrated along a warped extra dimension has been unknown. We study a five dimensional $SU(N)$ pure gauge theory with Randall-Sundrum warped compactification on S^1/Z_2. We clarify the notion of large gauge transformations that are non-periodic on the covering space for this setup. We obtain Kaluza-Klein expansions of gauge and ghost fields for the most general twists and background gauge field configurations, which break the gauge symmetry at classical level in general. We calculate the one-loop effective potential and find that the symmetry corresponding to the subgroup allowing continuous Wilson lines is dynamically restored. The presented method can be directly applied to include extra fields. The connection to dynamical Scherk-Schwarz supersymmetry breaking in warped space is discussed.

1 Introduction

Warped compactification not only provides a beautiful explanation how the large hierarchy $m_{\rm weak}/M_{\rm Planck}$ is generated from the exponential profile of its metric [1] but also is itself a quite general consequence of string theory due to the fact that D-branes generically provide sources for warping (See [2] and references therein). In the original Randall-Sundrum model [1], five dimensional spacetime is compactified on the orbifold S^1/Z_2, along which the normalization of the four dimensional metric is exponentially scaled. Orbifold compactification is a powerful tool in string theory to get three generations and especially to reduce the rank of the gauge group when combined with continuous Wilson lines [3]. The values of these Wilson lines and the resulting gauge symmetry breaking pattern must be determined dynamically via the Hosotani mechanism [4] when supersymmetry breaking is taken into account. Studies of Wilson line dynamics on simpler orbifolds have been started recently [5, 6]. These are also applied to gauge-Higgs unification models [7].

The orbifold compactification on S^1/Z_2 with radius $R = M_{\rm GUT}^{-1}$ provides a simple mechanism to solve the doublet-triplet splitting problem in $SU(5)$ grand unified theories (GUT's) [8]. Realistic models along this line have been proposed to implement low energy supersymmetry (SUSY) that is broken at the weak scale [9]. Once one considers an orbifold GUT, it is tempting to think that SUSY is also broken by the compactification via the Scherk-Schwarz mechanism [10]. However, in such an orbifold model the Scherk-Schwarz parameter, the amount of $SU(2)_R$ twist, must be put to an extremely small value of order $m_{\rm weak}/M_{\rm GUT}$ at the classical level [11]. Furthermore, quantum corrections lead either to restored supersymmetry $m_{\rm SUSY} = 0$ or to its violation of the order of $m_{\rm SUSY} \simeq R^{-1}$, unless one introduces an extra source of supersymmetry breaking at the orbifold fixed point [12]. This is generally true for symmetry breaking via the Hosotani mechanism in flat space.

Therefore it is natural to generalize the above considerations to a gauge theory in the bulk of the Randall-Sundrum geometry where one can make use of the exponential hierarchy in its metric. Warped SUSY GUT's are constructed in Refs. [13, 14] with SUSY breaking assumed to be by boundary conditions or by an extra source at the orbifold fixed point, respectively. Recently, it has been proposed in the framework of superconformal gravity that supersymmetry breaking by boundary conditions can be consistently defined and equivalent to Wilson line breaking, if the compensator multiplet has vanishing gauge coupling and the warping is generated from the vacuum configuration of the bulk scalars [15].[1] Eventually, the vacuum expectation value (vev) of the $SU(2)$ gauge field

[1]See also Ref. [16] for discussions on the Scherk-Schwarz breaking in the usual warped setup.

that is related to the Scherk-Schwarz twist must be determined dynamically by quantum corrections to the effective potential. To that end, it is important to solve the dynamics of Wilson lines in warped space, which has not been explored so far.[2]

In this paper we study the dynamics of Wilson lines of $SU(N)$ pure gauge theory in the bulk of Randall-Sundrum geometry. In the course of this calculation, we derive for the first time Kaluza-Klein expansions for the most general twists and background gauge field configurations and calculate the corresponding one-loop effective potential. In Section 2, we briefly review the Hosotani mechanism on the orbifold S^1/Z_2. In Section 3, we obtain the Kaluza-Klein (KK) expansions of the five dimensional gauge and ghost fields with most general twists in the presence of a gauge field background. In Section 4, we calculate the effective potential for the extra dimensional component of the background gauge field. In the last section we summarize and discuss our result.

2 Wilson lines on flat S^1/Z_2

We briefly review how twists and background gauge field configurations are related by large gauge transformations. We consider a five dimensional $SU(N)$ gauge theory compactified on the orbifold S^1/Z_2, which is obtained from the simply-connected space $R^1 : -\infty < y < \infty$ by modding with S^1 and Z_2 identifications $y \sim y + 2\pi R$ and $y \sim -y$, where R is the compactification radius. Under these identifications, the gauge fields A_M ($M = 0, \ldots, 3; 4$) are in general twisted by global $SU(N)$ transformations

$$\begin{aligned} A_M(-y) &= \pm P_0 A_M(y) P_0^{-1}, \\ A_M(\pi R + y) &= \pm P_1 A_M(\pi R - y) P_1^{-1}, \qquad A_M(y + 2\pi R) = U A_M(y) U^{-1}, \end{aligned} \tag{1}$$

where the extra $\pm$ sign is positive for four dimensions and negative for the extra dimension.[3] (We use μ for $0, \ldots, 3$ and y for both x^4 and index "4" such as $A_y = A_4$.) Note that the consistency conditions $U = P_1 P_0$ and $P_0^2 = P_1^2 = 1$ are imposed. Starting from the most general twists we can always choose the following basis [6]

$$\begin{aligned} P_0 &= \text{blockdiag}\,(\sigma_3, \quad \ldots, \quad \sigma_3, \quad I_r, \quad I_s, \quad -I_t, \quad -I_u), \\ P_1 &= \text{blockdiag}\,(\sigma_{\theta_1}, \quad \ldots, \quad \sigma_{\theta_q}, \quad I_r, \quad -I_s, \quad I_t, \quad -I_u), \end{aligned} \tag{2}$$

[2]In Ref. [17] a Wilson line in warped space is considered in the context of the AdS/Conformal Field Theory (CFT) correspondence, where the analysis is confined to a (potentially false) vacuum that corresponds to imposing both diagonal twists and vanishing background field configurations. See Ref. [24] for further discussions.

[3]In principle local identifications are possible but we assume them global for simplicity in this paper.

where I_r is $r\times r$ unit matrix, σ_a ($a=1,2,3$) are Pauli matrices, and $\sigma_\theta = \sigma_3\cos\theta+\sigma_1\sin\theta = e^{-i\theta\sigma_2}\sigma_3 = \sigma_3 e^{i\theta\sigma_2}$. ($2q+r+s+t+u=N$.) The A_y^a either within a block of $\pm I$ or connecting different blocks does not have a zero-mode, a mode having vanishing KK mass, and the dynamics of the corresponding Wilson line is trivial [18].[4] Therefore we can concentrate on a $SU(2)$ subblock with twists $P_0=\sigma_3$ and $P_1=\sigma_\theta$ without loss of generality. In general, only $A_y^{(2)}$ ($A_M = A_M^{(a)}\sigma_a/2$) has even Z_2 parities, hence a zero mode background: $gA_y^{(2)c}\equiv v$. The KK expansions are given by

$$A_\mu^{(2)}(x,y) = \sum_{n=1}^{\infty} A_{\mu n}^{(2)}(x)\frac{\sin\frac{ny}{R}}{\sqrt{\pi R}}, \qquad A_y^{(2)}(x,y) = A_{y0}^{(2)}(x)\frac{1}{\sqrt{2\pi R}} + \sum_{n=1}^{\infty} A_{yn}^{(2)}(x)\frac{\cos\frac{ny}{R}}{\sqrt{\pi R}}, \quad (3)$$

$$A_\mu^{\pm}(x,y) = \sum_{n=-\infty}^{\infty} A_{\mu n}(x)\frac{e^{\pm i(m_n+v)y}}{N_n}, \quad A_y^{\pm}(x,y) = \pm i\sum_{n=-\infty}^{\infty} A_{yn}(x)\frac{e^{\pm i(m_n+v)y}}{N_n}, \quad (4)$$

where $A_M^{\pm} = \frac{A_M^{(3)}\pm iA_M^{(1)}}{\sqrt{2}}$, $A_{Mn}(x)$ are real fields, $m_n = \frac{n}{R}+\frac{\theta}{\pi R}-v$ are KK masses, and $N_n = \sqrt{2\pi R}\, m_n$ are normalization constants. (Here we present a different form from literature in Eq. (4) to make the orthogonality of wave functions transparent.)

Consider the following large gauge transformation that is non-periodic on the covering space R^1:

$$igA_M(y) \to ig\widetilde{A}_M(y) = \left[\Omega\left(igA_M - \overleftarrow{\partial_M}\right)\Omega^{-1}\right](y) \quad \text{with} \quad \Omega(y) = \exp\left[i\varphi y\frac{\sigma_2}{2}\right], \quad (5)$$

which results in $g\widetilde{A}_y^{(2)c} = v - \varphi/\pi R$. We find that the shift of the background is canceled by the transformation of $A_M^{\pm} \to \widetilde{A}_M^{\pm} = e^{\mp i\varphi y/\pi R}A_M^{\pm}$ leaving its KK masses invariant: $\widetilde{m}_n = m_n$. Now the new fields are twisted by matrices $\widetilde{P}_0 = \sigma_3$ and $\widetilde{P}_1 = \sigma_{\theta-\varphi} \equiv \sigma_{\tilde\theta}$. Above, the two sets of twists P_i and $\widetilde{P}_i$ with the corresponding backgrounds are equivalent under the large gauge transformation, by which one may e.g. diagonalize the twist $\tilde\theta = 0$ or remove the background $\widetilde{A}_y^{(2)c} = 0$, but not both. If we choose to take the former (or latter) gauge, different values of $\widetilde{A}_y^{(2)c}$ (or $\tilde\theta$) correspond to physically different vacua which are degenerate at the classical level. Quantum corrections determine whether the symmetry is dynamically broken or restored, depending on the matter content [5, 6].

3 Kaluza-Klein expansions

We consider a $SU(N)$ gauge theory in the bulk of the Randall-Sundrum geometry [1], which is a five dimensional Anti de Sitter (AdS) space compactified on S^1/Z_2 with the

[4] Of course when, say, $r<u$, we can combine I_r and a part of $-I_u$ to form r additional σ_θ blocks with $\theta=0$.

metric:

$$G_{MN}dx^M dx^N = e^{-2\sigma(y)}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2, \tag{6}$$

where $\eta_{\mu\nu}$ is the Lorentzian metric and $\sigma(y)$ is defined by $\sigma(y) = k|y|$ at $-\pi R < y \le \pi R$, with k being the inverse AdS curvature radius. Elsewhere on the covering space R^1, we define σ by the periodicity condition $\sigma(y + 2\pi R) = \sigma(y)$. For later use we also define[5] $\epsilon(y) = \sigma'(y)/k$ and $z(y) = e^{\sigma(y)}$. We call the orbifold fixed points at $y = 0$ and $y = \pi R$ ultraviolet (UV) and infrared (IR) branes, respectively, and write $z_0 = z(0) = 1$ and $z_1 = z(\pi R) = e^{\pi k R}$. In this paper we assume that the radion is already stabilized e.g. by the Goldberger-Wise mechanism [19].

We employ the background field method, separating the gauge field into classical and quantum parts $A_M = A^c_M + A'_M$, and take the following gauge fixing[6]

$$S_f = -\frac{1}{\xi}\int d^4x \int_{-\pi R}^{\pi R} dy\sqrt{-G}\,\mathrm{tr}\,[ff]\,, \qquad \text{with} \quad f = z^2\eta^{\mu\nu}D^c_\mu A'_\nu + \xi z^t D^c_y z^{-t} A'_y, \tag{7}$$

where D_M is the gauge covariant derivative and the superscript c indicates that the gauge field is replaced by its classical part, e.g. $D^c_M A'_N = \partial_M A'_N + ig[A^c_M, A'_N]$. We consider the pure gauge background $F^c_{MN} = 0$, being a classical potential minimum, and assume $A^c_\mu = 0$ since it can be gauged away towards spatial infinity. When we choose $\xi = 1$ and $t = 2$, the quadratic terms for the gauge and ghost fields simplify:

$$S = \int d^4x \int_{-\pi R}^{\pi R} dy\,\mathrm{tr}\left[\eta^{\mu\nu}A'_\mu(\Box + \mathcal{P}_4)A'_\nu + A'_y z^{-2}(\Box + \mathcal{P}_y)A'_y + 2z^{-2}\overline{\omega}'(\Box + \mathcal{P}_4)\omega'\right], \tag{8}$$

where $\mathcal{P}_4 = D^c_y z^{-2} D^c_y$, $\mathcal{P}_y = D^c_y D^c_y z^{-2}$, and we have put $\omega^c = \overline{\omega}^c = 0$. (The surface terms vanish consistently for the given boundary conditions below.)

Following the same argument as in the flat case, we concentrate on a $SU(2)$ subblock with twists $P_0 = \sigma_3$ and $P_1 = \sigma_\theta$, without loss of generality. Again a zero mode resides only in $A^{(2)}_y$ and we can write $gA^{(2)c}_y(y) = vz^2$. (The derivation of the form of zero mode z^2 is given below.) To obtain the KK expansions, we follow the strategy of Ref. [20]. First, we solve the bulk KK equations at $0 < y < \pi R$ in terms of z neglecting all the boundary effects. Second, we put the boundary conditions at $z = z_0$ and z_1 on the obtained "downstairs"

[5] When there arises an ambiguity at the orbifold fixed point, say, around $y = 0$, we can use the regularized form $\sigma(y) = k\delta \log\cosh(y/\delta)$ with an infinitesimal $\delta = +0$ to check the expression. For our purpose we can use $\epsilon(y) = \theta(y) - \theta(-y)$, $\epsilon'(y) = 2\left[\delta(y) - \delta(y - \pi R)\right]$ and $\epsilon(y)^2 = 1$ at $-\pi R < y \le \pi R$.

[6] The choice $t = 4$ and $\xi = 1$ makes f manifestly invariant under five dimensional diffeomorphisms.

solution to make it consistent with the Z_2 twists so that the "upstairs" field on the covering space is well-defined, i.e. continuous everywhere.

Let us start with $A_\mu^{(2)}$ and $A_y^{(2)}$ which have definite odd and even Z_2 parities. We obtain the following KK expansions

$$A_\mu^{(2)}(x,y) = \sum_{n=1}^{\infty} A_{\mu n}^{(2)}(x)\epsilon \frac{z[J_1(\hat{M}_n z) + B_n Y_1(\hat{M}_n z)]}{N_n}, \quad A_y^{(2)}(x,y) = \sum_{n=0}^{\infty} A_{yn}^{(2)}(x)\frac{f_n(z)}{\mathcal{N}_n}, \quad (9)$$

where N_n, $\mathcal{N}_n$ are normalization constants, $B_n = -\frac{J_1(\hat{M}_n z_0)}{Y_1(\hat{M}_n z_0)} = -\frac{J_1(\hat{M}_n z_1)}{Y_1(\hat{M}_n z_1)}$, and the downstairs KK wave functions for vectoscalar $f_n(z) = z^2[J_0(\hat{M}_n z) + B_n Y_0(\hat{M}_n z)]$ are defined for later use. The KK masses $M_n = k\hat{M}_n$ are determined by zeros of the KK mass function: $J_1(\hat{M}z_1)Y_1(\hat{M}z_0) - Y_1(\hat{M}z_1)J_1(\hat{M}z_0)$, which we find is exactly the same for both $A_\mu^{(2)}$ and $A_y^{(2)}$.

In order to get the KK expansions of $A_M^{\pm}$, it is convenient to perform the large (background) gauge transformation (5), which again results in the new twists $\widetilde{P}_0 = \sigma_3$, $\widetilde{P}_1 = \sigma_{\theta-\varphi} \equiv \sigma_{\tilde{\theta}}$ and the background

$$g\widetilde{A}_y^{(2)c} = vz^2 - \frac{\varphi}{\pi R} = \left(v - \frac{2ka^2\varphi}{1-a^2}\right) z^2 - \frac{\varphi}{\pi R}\sum_{n=1}^{\infty}\frac{(1,f_n)}{\mathcal{N}_n^2} f_n(z), \quad (10)$$

where $a = z_0/z_1 = e^{-k\pi R} \ll 1$ and $(1,f_n) = \int_{z_0}^{z_1} 2\frac{dz}{kz} z^{-2} f_n(z)$. Unlike in the flat case, all the higher KK modes are generated, as can be seen in the last step of Eq. (10). However, these higher modes all vanish when integrated along the extra dimension in the Wilson line due to the orthogonality conditions of KK wave functions and one would expect that these modes can be gauged away.

To see this, consider the following background gauge transformation $\check{\Omega}(y) = \exp\left[i\mathcal{F}(y)\frac{\sigma_2}{2}\right]$, which we require to be normal in the sense that $\mathcal{F}(y)$ is continuous everywhere and periodic $\mathcal{F}(y+2\pi R) = \mathcal{F}(y)$. When $\mathcal{F}(y)$ is odd $\mathcal{F}(-y) = -\mathcal{F}(y)$, twists P_i are left invariant under this transformation, while gauge fields transform as $gA_y^{(2)} \to g\widetilde{A}_y^{(2)} = gA_y^{(2)} - \mathcal{F}'(y)$. Again, this shift is canceled by the transformation of $A_M^{\pm}$ in its KK mass. Let us take

$$\mathcal{F}(y) = k\sum_{n=1}^{\infty}\varphi_n F_n(y), \qquad \text{with} \quad F_n(y) = \int_0^y dy' f_n\left(z(y')\right), \quad (11)$$

where the summation is over all the non-zero modes. By definition, $F_n(y)$ is odd and its derivative is $f_n(y)$. Due to the downstairs boundary conditions, we find that $F_n(y)$ vanishes at both boundaries, i.e., the transformation $\check{\Omega}(y)$ is continuous everywhere on the covering space as well as periodic, as promised. To summarize, all the non-zero mode can always be removed by taking φ_n appropriately, without changing the twists P_i.

Now we choose $\varphi = \theta$ to diagonalize the twist $\tilde{\theta} = 0$ and gauge away all the resulting non-zero mode background. We then have definite Z_2 parity for all $\widetilde{A}_M^{(a)}$. Hereafter we omit the tilde for notational simplicity. The KK expansions are obtained as[7]

$$A_\mu^\pm(x,y) = \sum_{n=0}^{\infty} A_{\mu n}(x) E^{\pm i\epsilon \hat{v} z^2/2} \frac{\chi_{1,n}^\pm(z)}{N_n}, \quad A_y^\pm(x,y) = \sum_{n=0}^{\infty} A_{yn}(x)\epsilon E^{\pm i\epsilon \hat{v} z^2/2} \frac{\chi_{0,n}^\pm(z)}{\mathcal{N}_n}, \quad (12)$$

where $A_{Mn}(x)$ are real fields, $\hat{v} = v/k - 2\theta a^2/(1-a^2)$ is the dimensionless vev, $E^{\pm i\epsilon w} \equiv \cos w \pm i\epsilon \sin w$, N_n and $\mathcal{N}_n$ are normalization constants, and $\chi_{\nu,n}^\pm(z) = z^{2-\nu}[\alpha_n^\pm J_\nu(\hat{m}_n z) + \beta_n^\pm Y_\nu(\hat{m}_n z)]$ are the downstairs KK wave functions with $\hat{m}_n \equiv m_n/k$ and $\alpha_n^\pm, \beta_n^\pm$ being dimensionless KK masses and complex constants, respectively. (Note $\alpha_n^\pm = \frac{\alpha_n^{(3)} \pm i\alpha_n^{(1)}}{\sqrt{2}}$ etc.) The boundary conditions on the downstairs fields can be summarized as

$$M(\hat{m}_n)\vec{V} = 0, \quad (13)$$

with $\vec{V} = (\alpha_n^{(3)}\ \beta_n^{(3)}\ \alpha_n^{(1)}\ \beta_n^{(1)})^T$ for A_μ, $\vec{V} = (\alpha_n^{(1)}\ \beta_n^{(1)}\ \alpha_n^{(3)}\ \beta_n^{(3)})^T$ for A_y, and

$$M(\hat{m}_n) = \begin{pmatrix} \mathcal{J}_C(1) & \mathcal{Y}_C(1) & \mp\mathcal{J}_S(1) & \mp\mathcal{Y}_S(1) \\ \mathcal{J}_C(0) & \mathcal{Y}_C(0) & \mp\mathcal{J}_S(0) & \mp\mathcal{Y}_S(0) \\ \pm\sin\frac{\hat{v}z_1^2}{2} J_\nu(\hat{m}_n z_1) & \pm\sin\frac{\hat{v}z_1^2}{2} Y_\nu(\hat{m}_n z_1) & \cos\frac{\hat{v}z_1^2}{2} J_\nu(\hat{m}_n z_1) & \cos\frac{\hat{v}z_1^2}{2} Y_\nu(\hat{m}_n z_1) \\ \pm\sin\frac{\hat{v}z_0^2}{2} J_\nu(\hat{m}_n z_0) & \pm\sin\frac{\hat{v}z_0^2}{2} Y_\nu(\hat{m}_n z_0) & \cos\frac{\hat{v}z_0^2}{2} J_\nu(\hat{m}_n z_0) & \cos\frac{\hat{v}z_0^2}{2} Y_\nu(\hat{m}_n z_0) \end{pmatrix},$$

where upper and lower signs as well as $\nu = 1$ and 0 are for A_μ and A_y, respectively, $\begin{pmatrix} \mathcal{J}_C(i) \\ \mathcal{J}_S(i) \end{pmatrix} = \begin{pmatrix} \cos\frac{\hat{v}z_i^2}{2} & -\sin\frac{\hat{v}z_i^2}{2} \\ \sin\frac{\hat{v}z_i^2}{2} & \cos\frac{\hat{v}z_i^2}{2} \end{pmatrix} \begin{pmatrix} \nu J_\nu(\hat{m}z_i) + \hat{m}_n z_i J_\nu'(\hat{m}z_i) \\ \hat{v}z_i^2 J_\nu(\hat{m}z_i) \end{pmatrix}$, and $\mathcal{Y}_C$ and $\mathcal{Y}_S$ are defined similarly to $\mathcal{J}_C$ and $\mathcal{J}_S$ with J replaced by Y. (J_ν and Y_ν are Bessel functions of order ν.) The KK mass function for $A^\pm$ is obtained from the determinant of the boundary condition matrix:

$$N(\hat{m}) = \det M(\hat{m}), \quad (14)$$

and $N(\hat{m}) = 0$ determines the KK masses. We find that the N's are exactly the same for A_μ and A_y and that its dependence on $\hat{v}$ is only through the term $\frac{2}{\pi^2}\cos[\hat{v}(z_1^2 - z_0^2)]$. The

[7] To derive Eq. (12) we have used $\epsilon^2 = 1$ in $A_y^{(1)}$, which can be justified similarly as above.

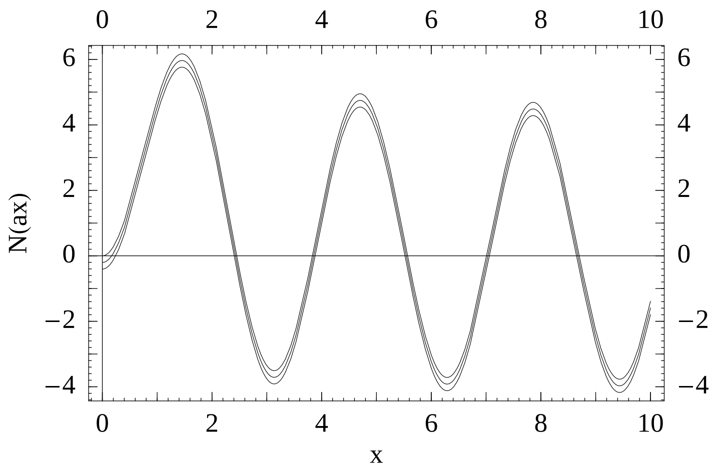

Figure 1: KK mass function $N(ax)$ vs x with $a = 10^{-15}$ for $\hat{v}/a^2 = 0, \frac{\pi}{2}, \pi$. The zeros of N correspond to the KK masses. Note that a massless mode appears only for $\hat{v} = 0$ mod 2π.

KK expansions for the ghost field is obtained similarly to A_μ. (Recall that the antighost field $\overline{\omega}$ is not necessarily the complex conjugate of the ghost field ω.)

In Fig. 1 we plot $N(ax)$ as a function of $x = \hat{m}/a$ within half a period of $\hat{v}$, i.e. for $\hat{v}(z_1^2 - z_0^2) = 0, \pi/2, \pi$ from above to below when $a = 10^{-15}$. The points where the curve crosses the x-axis give the values of the corresponding KK masses, with even and odd modes appearing in alternating order. We can see that there appears an extra massless mode for $\hat{v} = 0$, as expected, and that the dependence on the variation of $\hat{v}$ is strongest for this would-be zero mode, whose mass can be easily determined from Eq. (14) to be

$$m_0 = ka\sqrt{\frac{1 - \cos(\hat{v}/a^2)}{k\pi R}}, \tag{15}$$

in the approximation $a \ll 1$. For the maximal breaking with $\cos(\hat{v}/a^2) = -1$, we find $m_0 \approx 0.24\, ka$ for $a = 10^{-15}$.

4 One loop effective potential

Following convention, we perform the dimensional reduction in the coordinate frame where the warp factor is unity at the UV brane.[8] The contribution of a pair of Z_2 even and odd

[8]The physical effective potential from the point of view of the IR brane will be enhanced by a^{-4} [21].

gauge fields $A_M^{(3)}$ and $A_M^{(1)}$ is

$$V_{\rm eff}=\frac{\mu^{4-d}}{2}\int\frac{d^dp}{(2\pi)^d}\sum_{n=0}^{\infty}\log(p^2+m_n^2)=-\frac{1}{2}\frac{(ka)^4}{(4\pi)^2}\left(\frac{4\pi\mu^2}{k^2a^2}\right)^{\varepsilon/2}\Gamma\left(-2+\frac{\varepsilon}{2}\right)\sum_{n=0}^{\infty}x_n^{4-\varepsilon},\quad(16)$$

per degree of freedom, where $d=4-\varepsilon$ is the number of dimensions with ε being infinitesimal, μ is an arbitrary scale, and $x_n=m_n/ka$ is the dimensionless KK mass. The infinite sum over KK masses can be evaluated utilizing zeta function regularization techniques [21, 22, 23]

$$v_{\rm eff}(\hat{v})\equiv-\Gamma\left(-2+\frac{\varepsilon}{2}\right)\sum_{n=0}^{\infty}x_n^{4-\varepsilon}=-\Gamma\left(-2+\frac{\varepsilon}{2}\right)\int_C\frac{dx}{2\pi i}x^{4-\varepsilon}\frac{N'(ax)}{N(ax)},\quad(17)$$

where C is a contour encircling all the poles on the positive real axis counter-clockwise. Note that these are the only poles in the right half plane since there is a one-to-one correspondence between the zeros of the KK mass function (14) and the eigenvalues of the operators $\mathcal{P}_4$ and $\mathcal{P}_y$ in Eq. (8) which are Hermitian with respect to our boundary conditions.

After a few manipulations, we find

$$\begin{aligned}v_{\rm eff}(\hat{v})=&\,I_{\rm IR}+\frac{I_{\rm UV}}{a^{4-\varepsilon}}+2\int_0^\infty dx\,x^{3-\varepsilon}\log\Bigg[\\&1-\frac{1}{2}\left(\frac{K_0(x)I_0(ax)}{I_0(x)K_0(ax)}+\frac{K_1(x)I_1(ax)}{I_1(x)K_1(ax)}-\frac{K_0(x)I_1(ax)}{I_0(x)K_1(ax)}-\frac{K_1(x)I_0(ax)}{I_1(x)K_0(ax)}\right)\\&+\frac{K_0(x)K_1(x)I_0(ax)I_1(ax)}{I_0(x)I_1(x)K_0(ax)K_1(ax)}-\frac{\cos\left(\frac{\hat{v}}{a^2}(1-a^2)\right)}{2ax^2I_0(x)I_1(x)K_0(ax)K_1(ax)}\Bigg],\\\simeq&\,I_{\rm IR}+\frac{I_{\rm UV}}{a^{4-\varepsilon}}+2\int_0^\infty dx\,x^{3-\varepsilon}\log\left[1-\frac{I_0(x)K_1(x)-K_0(x)I_1(x)-\frac{1}{x}\cos\frac{\hat{v}}{a^2}}{2I_0(x)I_1(x)\left(\gamma+\log\frac{ax}{2}\right)}\right],\end{aligned}\quad(18)$$

where divergent integrals $I_{\rm IR}$ and $I_{\rm UV}$ are independent of v and a and can be absorbed in the renormalization of the IR- and UV-brane tensions, respectively, as in Ref. [23]. (I_ν and K_ν are the modified Bessel functions.) We find that the effective potential is a periodic function of $\hat{v}$ with the period $2\pi a^2/(1-a^2)$ as is expected from the shape of the KK mass function. In the last line of Eq. (18), the small a limit is taken, assuming that $\hat{v}$ is within the first period, i.e. $\hat{v}/a^2=O(1)$, without loss of generality.[9] (One might find it suggestive that the scale of the period of v is of the order of ka^2, which roughly corresponds to the order of the observed value of the cosmological constant $\simeq$ meV.)

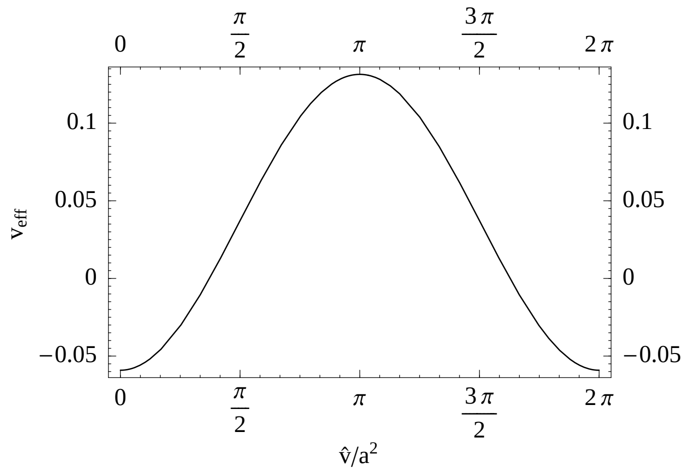

Figure 2: Normalized effective potential v_{eff} vs $\hat{v}/a^2$ with $a = 10^{-15}$.

In Fig. 2, we plot v_{eff} as a function of $\hat{v}/a^2$ when $a = 10^{-15}$. Contribution from the ghost loop is equal to Eq. (18) multiplied by -2. The final result including gauge and ghost field contributions is therefore:

$$V_{\text{eff}} = \frac{3}{32\pi^2}(ka)^4 v_{\text{eff}}. \tag{19}$$

Note that if we include extra adjoint bulk fermions, they would contribute with opposite signs to Eq. (18) and that if we add more than required to make the theory supersymmetric, the potential of Fig. 2 would be flipped upside down, realizing a dynamical symmetry breaking vacuum which corresponds to the maximal twist $\theta = \pi/2$ in the $A_y^c = 0$ gauge. This vacuum breaks $SU(2)$ completely and hence provides a rank reduction of the gauge symmetry. We find that the symmetry breaking scale is of the order of $ka \simeq$ TeV for this case.

5 Summary and discussions

We have studied the $SU(N)$ pure gauge theory in the bulk of the Randall-Sundrum geometry and have obtained Kaluza-Klein expansions of gauge and ghost fields under the presence of the gauge field background with most general twists P_i. We find that four dimensional gauge, vectoscalar and ghost fields have exactly the same KK masses. During

[9]For large x, the integrand goes to zero and a small a expansion can be performed with converging coefficients.

the course of this calculation we have clarified the notion of a large gauge transformation that is non-periodic on the covering space and how it is consistently realized in the warped background. The effective potential for the background A_y^c is obtained. We find that a gauge symmetry corresponding to a continuous Wilson line, i.e. a $SU(2)$ subgroup of Eq. (2), which is completely broken for finite θ at the classical level, is dynamically restored to $U(1)$.

It is straightforward to apply our method to include other fields with or without extra boundary masses and especially to supersymmetrize our setup, where the symmetry breaking scales due to continuous Wilson lines will be of the order of $ka \simeq$ TeV according to the analysis presented here. Since we find that the dynamics of Wilson lines in warped space can be controlled in quite a parallel manner to that in flat space if one treats the large gauge transformation carefully, we expect that the Hosotani mechanism will lead to a dynamical supersymmetry breaking of the order of weak scale in the setup discussed in the introduction. It is also interesting to pursue the AdS/CFT correspondence generalizing the analysis of Ref. [17] to our setup, as the Wilson line on the AdS side will correspond to a quantity that is integrated all the way from UV to IR on the CFT side. The techniques developed here can also be applied to the gauge-Higgs unification models in warped space. These points will be presented in separate publications [24].

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